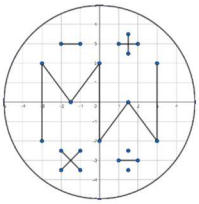

Mr. Wright's Math Extravaganza

Precalculus
Matrices

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications

Score	I Can Statements
4.0	☐ I can demonstrate in-depth inferences and applications that go beyond what was taught.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content
3.0	☐ I can solve a system of linear equations using Gauss-Jordan elimination. ☐ I can solve a system of linear equations using an inverse matrix.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content
2.0	☐ I can put a matrix in row-echelon form and reduced-row-echelon form. ☐ I can perform matrix operations ($+$, $-$, $\times$). ☐ I can find the inverse of a square matrix. ☐ I can find the determinant of a matrix.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

Precalculus

9-01 Matrices and Systems of Equations

Matrix

- Rectangular _____of numbers

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

- $a_{row,column}$
- Each entry is an _____
- Augmented Matrix
  - Two matrices _____together
- Order of matrix
  - _____
  - _____× _____

What is the order of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$?

Elementary Row Operations

- **_____2 rows**
- **_____a row by a nonzero constant**
- **_____a multiple of a row to another row**

Add 2 times 1st row to the 2nd row: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Row-Echelon Form

- All rows consisting entirely of _____are at _____
- For other rows, the first _____entry is _____
- For successive rows, the leading 1 in the _____row is farther to the _____
- $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Reduced Row-Echelon Form

- Columns with leading 1 have _____ as other entries

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve $\begin{cases} x + 3y + 4z = 7 \\ 2x + 7y + 5z = 10 \\ 3x + 10y + 4z = 27 \end{cases}$

Precalculus

9-02 Gaussian Elimination

Gaussian Elimination

- Solving a system of linear equations by putting it into _____form with elementary row operations

Gauss-Jordan Elimination

- Solve by putting the system into _____row-echelon form
- If a row becomes all zeros with final entry not zero = _____solution
- If a row becomes all zeros = _____solutions

Solve $\begin{cases} x - 3z = -5 \\ 3x + y - 2z = -4 \\ 2x + 2y + z = -2 \end{cases}$

Solve $\begin{cases} x + y + 5z = -3 \\ -x - 2y - 8z = 5 \\ -x - 2z = 1 \end{cases}$

Precalculus

9-03 Matrix Operations

Matrix addition and subtraction

- Both matrices must have same _____
- Add or subtract _____elements

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \\ -4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -2 & -3 \\ -4 & -5 \end{bmatrix}$$

Scalar multiplication

- Multiply a matrix with a _____
- _____

$$3\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \end{bmatrix}$$

Matrix multiplication

- Number of _____in 1st = number of _____in 2nd

$$(m \times n) \cdot (n \times p)$$

- Order of product _____
- Order is _____
- **NO COMMUTATIVE PROPERTY!!!!!**

$$\begin{bmatrix} 2 & -1 & 7 \\ 0 & 6 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & 4 \\ -2 & 1 & 2 \end{bmatrix}$$

Precalculus

9-04 Inverse Matrices

Identity Matrix (*I*)

- $A \cdot I = A$
- $A \cdot A^{-1} = I$
- Both A and A^{-1} must be _____
- $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ OR $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ OR $I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Inverse of 2×2

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$**, then**

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find the inverse of $\begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$

Find other inverses

- _____the matrix with the _____matrix
- Use Gauss-Jordan elimination to turn the _____matrix into the _____matrix
- $[A \vdots I] \rightarrow [I \vdots A^{-1}]$

Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ -3 & 4 & -4 \end{bmatrix}$

Use an inverse to solve system of equations

- Write system as _____
- $AX = B$ (coefficients · variables = constants)
- $A^{-1}AX = A^{-1}B$
- $IX = A^{-1}B$
- $X = A^{-1}B$
- Solve by multiplying the _____of the coefficients with the _____

Solve $\begin{cases} 2x + 3y = 0 \\ x - 4y = 7 \end{cases}$

Precalculus

9-05 Determinants of Matrices

Determinant

- _____number associated with a _____matrix

2×2

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$
$$= ad - bc$$

- _____product – _____ product

Find $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

3×3

- Copy 1st _____columns _____matrix
- + products of _____– products of _____

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Expansion by cofactors

- Sign Pattern

$$\begin{bmatrix} + & - & + & - & \cdots \\ - & + & - & + & \cdots \\ + & - & + & - & \cdots \\ - & + & - & + & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

- Minor
  - Determinant of matrix created by crossing out a _____and _____
- Cofactor
  - Minor with _____from sign pattern

Given $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$, find

Minor M_{13} Cofactor C_{13}

Find $\begin{vmatrix} -1 & 0 & 4 \\ 3 & -2 & 0 \\ 1 & -1 & 1 \end{vmatrix}$

Find $\begin{vmatrix} -2 & 4 & 0 & 5 \\ 0 & 2 & -1 & 0 \\ 3 & 1 & -4 & -1 \\ -5 & 0 & -2 & 3 \end{vmatrix}$

Precalculus

9-06 Applications of Matrices

Cramer's Rule

- Used to solve _____
- $x_1 = \frac{|A_1|}{|A|}$ $x_2 = \frac{|A_2|}{|A|}$
- A = _____matrix
- A_n = _____matrix with column n replaced with _____
- If $|A| = 0$, then _____solution or _____solutions

Use Cramer's Rule $\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ y - 2z = -3 \end{cases}$

Area of triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3)

$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Find the area of triangle with vertices $(-3, 1)$, $(2, 4)$, $(5, -3)$

Lines in a Plane

- If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$, then the points are collinear

Find equation of line given 2 points (x_1, y_1) and (x_2, y_2)

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Find the equation of the line passing through $(-2, 9)$ and $(3, -1)$

Hill Cypher Encoding a Message

1. Convert the message into _____
2. Choose a _____encoding matrix.
3. _____the message numbers into matrices of 1 row and the same number of _____as the encoding matrix.
4. _____the letter matrices with the encoding matrix.
5. The encoded message is the list of _____produced.
6. Decode by using _____of encoding matrix

_ = 0	I = 9	R = 18
A = 1	J = 10	S = 19
B = 2	K = 11	T = 20
C = 3	L = 12	U = 21
D = 4	M = 13	V = 22
E = 5	N = 14	W = 23
F = 6	O = 15	X = 24
G = 7	P = 16	Y = 25
H = 8	Q = 17	Z = 26

Encode LUNCH using $\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$