Precalculus

9-01 Matrices and Systems of Equations

Matrix

- Rectangular _____of numbers

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

- $a_{row,column}$
- Each entry is an _____
- Augmented Matrix
  - Two matrices _____together
- Order of matrix
  - _____
  - _____× _____

What is the order of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$?

Elementary Row Operations

- **_____2 rows**
- **_____a row by a nonzero constant**
- **_____a multiple of a row to another row**

Add 2 times 1st row to the 2nd row: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Row-Echelon Form

- All rows consisting entirely of _____are at _____
- For other rows, the first _____entry is _____
- For successive rows, the leading 1 in the _____row is farther to the _____
- $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Reduced Row-Echelon Form

- Columns with leading 1 have _____ as other entries

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve $\begin{cases} x + 3y + 4z = 7 \\ 2x + 7y + 5z = 10 \\ 3x + 10y + 4z = 27 \end{cases}$

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9-02 Gaussian Elimination

Gaussian Elimination

- Solving a system of linear equations by putting it into _____form with elementary row operations

Gauss-Jordan Elimination

- Solve by putting the system into _____row-echelon form
- If a row becomes all zeros with final entry not zero = _____solution
- If a row becomes all zeros = _____solutions

Solve $\begin{cases} x - 3z = -5 \\ 3x + y - 2z = -4 \\ 2x + 2y + z = -2 \end{cases}$

Solve $\begin{cases} x + y + 5z = -3 \\ -x - 2y - 8z = 5 \\ -x - 2z = 1 \end{cases}$

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9-03 Matrix Operations

Matrix addition and subtraction

- Both matrices must have same _____
- Add or subtract _____elements

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \\ -4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -2 & -3 \\ -4 & -5 \end{bmatrix}$$

Scalar multiplication

- Multiply a matrix with a _____
- _____

$$3\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \end{bmatrix}$$

Matrix multiplication

- Number of _____in 1st = number of _____in 2nd

$$(m \times n) \cdot (n \times p)$$

- Order of product _____
- Order is _____
- **NO COMMUTATIVE PROPERTY!!!!!**

$$\begin{bmatrix} 2 & -1 & 7 \\ 0 & 6 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & 4 \\ -2 & 1 & 2 \end{bmatrix}$$

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9-04 Inverse Matrices

Identity Matrix (*I*)

- $A \cdot I = A$
- $A \cdot A^{-1} = I$
- Both A and A^{-1} must be _____
- $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ OR $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ OR $I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Inverse of 2×2

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$**, then**

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find the inverse of $\begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$

Find other inverses

- _____the matrix with the _____matrix
- Use Gauss-Jordan elimination to turn the _____matrix into the _____matrix
- $[A \vdots I] \rightarrow [I \vdots A^{-1}]$

Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ -3 & 4 & -4 \end{bmatrix}$

Use an inverse to solve system of equations

- Write system as _____
- $AX = B$ (coefficients · variables = constants)
- $A^{-1}AX = A^{-1}B$
- $IX = A^{-1}B$
- $X = A^{-1}B$
- Solve by multiplying the _____of the coefficients with the _____

Solve $\begin{cases} 2x + 3y = 0 \\ x - 4y = 7 \end{cases}$

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9-05 Determinants of Matrices

Determinant

- _____number associated with a _____matrix

2×2

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$
$$= ad - bc$$

- _____product – _____ product

Find $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

3×3

- Copy 1st _____columns _____matrix
- + products of _____– products of _____

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Expansion by cofactors

- Sign Pattern

$$\begin{bmatrix} + & - & + & - & \cdots \\ - & + & - & + & \cdots \\ + & - & + & - & \cdots \\ - & + & - & + & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

- Minor
  - Determinant of matrix created by crossing out a _____and _____
- Cofactor
  - Minor with _____from sign pattern

Given $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$, find

Minor M_{13} Cofactor C_{13}

Find $\begin{vmatrix} -1 & 0 & 4 \\ 3 & -2 & 0 \\ 1 & -1 & 1 \end{vmatrix}$

Find $\begin{vmatrix} -2 & 4 & 0 & 5 \\ 0 & 2 & -1 & 0 \\ 3 & 1 & -4 & -1 \\ -5 & 0 & -2 & 3 \end{vmatrix}$

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9-06 Applications of Matrices

Cramer's Rule

- Used to solve _____
- $x_1 = \frac{|A_1|}{|A|}$ $x_2 = \frac{|A_2|}{|A|}$
- A = _____matrix
- A_n = _____matrix with column n replaced with _____
- If $|A| = 0$, then _____solution or _____solutions

Use Cramer's Rule $\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ y - 2z = -3 \end{cases}$

Area of triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3)

$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Find the area of triangle with vertices $(-3, 1)$, $(2, 4)$, $(5, -3)$

Lines in a Plane

- If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$, then the points are collinear

Find equation of line given 2 points (x_1, y_1) and (x_2, y_2)

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Find the equation of the line passing through $(-2, 9)$ and $(3, -1)$

Hill Cypher Encoding a Message

1. Convert the message into _____
2. Choose a _____encoding matrix.
3. _____the message numbers into matrices of 1 row and the same number of _____as the encoding matrix.
4. _____the letter matrices with the encoding matrix.
5. The encoded message is the list of _____produced.
6. Decode by using _____of encoding matrix

_ = 0	I = 9	R = 18
A = 1	J = 10	S = 19
B = 2	K = 11	T = 20
C = 3	L = 12	U = 21
D = 4	M = 13	V = 22
E = 5	N = 14	W = 23
F = 6	O = 15	X = 24
G = 7	P = 16	Y = 25
H = 8	Q = 17	Z = 26

Encode LUNCH using $\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$